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Qubit portraits of qudit states and quantum correlations

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Abstract

The machinery of qubit portraits of qudit states, recently presented, is considered here in more details in order to characterize the presence of quantum correlations in bipartite qudit states. In the tomographic representation of quantum mechanics, Bell-like inequalities are interpreted as peculiar properties of a family of classical joint probability distributions which describe the quantum state of two qudits. By means of the qubit-portraits machinery a semigroup of stochastic matrices can be associated with a given quantum state. The violation of the CHSH inequalities is discussed in this framework with some examples; we found that quantum correlations in qutrit isotropic states can be detected by the suggested method while it cannot be in the case of qutrit Werner states.

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1. Introduction

Entanglement is probably one of the most intriguing and fascinating characteristics of quantum mechanics [1]; its importance lies at the heart of the physical interpretation of the theory. The scientific interest and efforts toward the understanding and a complete characterization of entanglement are motivated both by its role in the conceptual foundation of quantum theory and by all the recent proposals and applications which lead us to consider entanglement as a resource for quantum information and computation tasks [2].

Although the two concepts are not equivalent, the presence of entanglement is strongly related to quantum non-locality. The fundamental tools to study quantum non-locality, i.e. quantum correlations, are the Bell-like inequalities. A violation of a Bell-like inequality is an evidence of the presence of non-local correlations in the quantum state. It is well known that only entangled states can violate Bell-like inequalities. In the present paper we study bipartite mixed states entanglement by looking at violations of a Bell-like inequality; to do this we exploit the point of view given by the tomographic description of quantum mechanics [3].

The main goal of the present contribution is to further analyze the linear map which defines the *qubit portraits* of the qudit state introduced in [4]. In particular we consider how this map can be used to describe quantum correlation in a bipartite quantum system. This paper has two main ingredients: the first is the tomographic description of quantum mechanics; the second is related to the CHSH inequalities [5, 6]. The tomographic approach is known to be mathematically equivalent to the other descriptions of quantum mechanics based, for instance, on density matrices or Wigner functions. Nevertheless there are two conceptually relevant differences: the first one is that in the tomographic approach one deals only with well-defined (classical) probability distributions which are directly related to experimentally accessible relative frequencies of measurement outcomes; the second one is that, in order to define a tomogram, one needs additional information about the observables related to a given experimental setup. From these considerations it can be argued that the tomographic approach can be viewed as a rather natural framework to study Bell-like inequalities. In the present paper we study the well-known CHSH inequalities in this framework. Although the tomographic description of quantum mechanics can be defined in full generality [7], here we focus our attention on quantum systems with finite levels.

Among a plethora of proposed criteria to detect entanglement, a prominent position is held by a family of methods which are based on the action of special linear maps on the set of separable quantum states. Examples are the criteria based on positive but not completely positive maps [8] (like the criterion of the positive partial transpose [9]) and the realignment criterion [10] which can be understood from a unique point of view based on linear contractions [11]. Another example is given by the criterion based on partial scaling transform [12] which is a linear map that is neither completely positive nor positive. In the present paper we make use of the qubit portraits of a qudit state [4] which is again a linear map but is defined in the tomographic description of quantum mechanics.

The paper is organized in the following way. In section 2 we briefly recall some definitions and basic properties about tomograms. In section 3 the CHSH inequalities are presented in the framework of the tomographic approach to quantum mechanics. In section 4 the machinery of qubit portraits of a qudit system is considered in order to deal with higher dimensional systems, examples for qubit and qutrit Werner and isotropic states are presented. The paper ends with final remarks and conclusions in section 6.

2. Introduction to quantum tomograms

Let us consider a d-level quantum system with the associated Hilbert space $\mathcal{H} \cong C^d$ and a chosen basis $\{|m\rangle\}_{m=1,\ldots,d}$. Given a state of a system expressed by means of a density operator ρ, there are several ways to define a corresponding tomogram; let us first consider the definition of a *unitary tomogram*. The diagonal elements $\langle m|\rho|m\rangle$ of the density operator are the populations in the given basis, they constitute a well-defined probability distribution. The knowledge of the populations in a given basis is in general not sufficient to reconstruct the off-diagonal elements of the density operator, on the other hand the knowledge of the populations in all possible bases gives a complete information about the quantum state of the system. As the unitary group acts transitively on the family of bases, a generic basis $\{|m'\rangle\}$ can be identified with a special unitary transformation $u \in \mathrm{SU}(d)$ with $|m'\rangle = u|m\rangle$. These considerations yield the definition of the unitary tomogram as follows:

$$\omega_\rho(m, u) \equiv \langle m|u^\dagger \rho u|m\rangle. \tag{1}$$

The tomogram is thus a family of well-defined probability distributions over d possible measurement outcomes, which depends on the $d^2 - 1$ parameters defining a special-unitary transformation. It is thus apparent that the tomogram explicitly gives the probability distributions for the outcomes of all the possible projective measurements allowed by the principles of quantum mechanics. As a matter of fact this is a redundant description; a lower number of bases would be sufficient as long as they constitute a tomographic set [7].

Let us now consider a special case, in which the d-level system is indeed a spin-j particle, with $d = 2j + 1$, and the state vectors belonging to the basis are eigenstates of the angular momentum along a quantization axis, say $\hat{z}$. In this case, one can be mostly interested in measurements of polarization along a generic direction $\hat{n}$. Hence one is led to define the *spin tomogram* as follows:

$$\omega_\rho^j(m, D) \equiv \langle m|D^\dagger \rho D|m\rangle, \tag{2}$$

where D belongs to a spin-j irreducible representation of the group SU(2) and has the following expression (see [14], for instance):

$$\langle m'|D|m\rangle = \mathrm{e}^{-\mathrm{i}m'\phi} d^j_{m'm}(\theta)\, \mathrm{e}^{-\mathrm{i}m\gamma}, \tag{3}$$

where

$$d^j_{m'm}(\theta) = \left[\frac{(j+m)!(j-m)!}{(j+m')!(j-m')!}\right]^{1/2} \left(\sin\frac{\theta}{2}\right)^{m-m'} \left(\cos\frac{\theta}{2}\right)^{m+m'} P_{j-m}^{(m-m',m+m')}(\cos\theta) \tag{4}$$

is the Wigner matrix and $P_{j-m}^{(m-m',m+m')}$ are the Jacobi polynomials. A unitary operator D is uniquely identified by the three Euler angles, nevertheless since only the diagonal elements of the (rotated) density operator appear in the definition, the tomogram depends only on two Euler angles, say θ and ϕ, or equivalently on a point on the Bloch sphere $\hat{n} \equiv (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. Note that both kinds of tomograms are mathematically equivalent to the density matrix description of the quantum states. In the case of a spin tomography an additional physical information is added; this information allows us to restrict to bases generated by an irreducible representation of SU(2) acting on a properly chosen fiducial one.

Let us study quantum entanglement in the tomographic picture (see also some aspects of this approach in [13]). In order to set properly the problem of separability of a quantum state, one needs primarily to identify a partition of the whole system into a number of subsystems each of dimension d_k. This can be done mathematically with the only constraint that $\Pi d_k = d$, nevertheless the definition of subsystems is in general physically determined and depends on the experimentally achievable observables and operations. To fix the ideas, let us for instance consider the case of a spin-j particle which turns to be a bipartite system composed of a spin-j_1 and spin-j_2, with $d_k = 2j_k + 1$ and $d_1 d_2 = d$. It is natural to define another kind of tomogram, which we call *local spin tomogram*, as follows:

$$\omega_\rho^{j_1 j_2}(m_1, m_2, D_1, D_2) \equiv \langle m_1 m_2|D_1^\dagger \otimes D_2^\dagger \rho D_1 \otimes D_2|m_1 m_2\rangle, \tag{5}$$

where $m_k = -j_k, -j_k+1, \ldots, j_k$, and D_k are unitary irreducible representations of SU(2). An analogous construction can be made for *local unitary tomography*, which yields the definition:

$$\omega_\rho(m_1, m_2, u_1, u_2) \equiv \langle m_1 m_2|u_1^\dagger \otimes u_2^\dagger \rho u_1 \otimes u_2|m_1 m_2\rangle. \tag{6}$$

These definitions may be immediately extended to the multi-partite case. Note that, while in the density matrix description the information about the internal structure of the system has to be inserted as an additional information, in the tomographic approach it is included in the chosen kind of tomogram from the very beginning. In the case of local spin tomography (5) the tomogram is a family of probability distributions depending on the two pairs of Euler angles (θ_k, ϕ_k) which determine the directions of polarization $\hat{n}_1$ and $\hat{n}_2$ for the first and second particle, respectively.

Since the tomogram is a family of well-defined probability distributions we find that the tomographic approach to quantum mechanics can be a natural candidate to deal with quantum probabilities and correlations and also to study violation of Bell-like inequalities. Let us consider an observable X; it identifies a preferred basis $|\bar{m}\rangle = \bar{u}|m\rangle$ in terms of its eigenstates; then the expectation value is simply written as

$$\langle X\rangle_\rho = \sum_m x_m \omega_\rho(m, \bar{u}), \tag{7}$$

where x_m are the corresponding eigenvalues. Let us consider the case of a bipartite system with a couple of local observables X_1 and X_2 with the corresponding eigenstates $|\bar{m}_k\rangle = \bar{u}_k|m_k\rangle$ and eigenvalues $x_{m,k}$. In the tomographic picture the correlation $C_\rho(X_1, X_2) = \langle X_1 X_2\rangle_\rho$ is written as follows:

$$C_\rho(X_1, X_2) = \sum_{m_1, m_2} x_{m_1,1} x_{m_2,2} \omega_\rho(m_1, m_2, \bar{u}_1, \bar{u}_2). \tag{8}$$

Given a bipartite system with a simply separable density operator $\rho = \rho_1 \otimes \rho_2$ it follows from the definitions (5) or (6) that

$$\omega_\rho(m_1, m_2, u_1, u_2) = \omega_{\rho_1}(m_1, u_1)\omega_{\rho_2}(m_2, u_2), \tag{9}$$

that is, the tomogram itself is the product of two tomograms and, in particular, it defines a family of uncorrelated joint probability distributions. By linearity, it follows that a generic separable state with the density matrix $\rho = \sum_k p_k \rho_1^k \otimes \rho_2^k$ has a tomogram of the form

$$\omega_\rho(m_1, m_2, u_1, u_2) = \sum_k p_k \omega_{\rho_1^k}(m_1, u_1)\omega_{\rho_2^k}(m_2, u_2), \tag{10}$$

which corresponds to a family of probability distributions with (classical) correlations. Note that the tomogram is a family of well-defined *classical* probability distributions in *any* case, for separable states the decomposition (10) exists with constant $p_k \geqslant 0$ and $\omega_{\rho_1^k}(m_1, u_1)$ and $\omega_{\rho_2^k}(m_2, u_2)$ which are well-defined tomograms.

3. CHSH inequalities in the tomographic picture

In this section we review the CHSH inequalities exploiting the tomographic description of quantum mechanics and quantum correlations. In order to do this, we introduce a stochastic matrix which is determined by a given tomogram whose structure is related to the form of the CHSH inequalities. These inequalities were introduced in [6] as a generalization of the original Bell's inequalities [5] in order to relax some experimentally unfeasible assumptions. The setting in which the inequalities are formulated is made by an ensemble of pairs of correlated particles moving in opposite directions and entering respectively two measurement apparatuses, say I_a and II_b, where a and b are adjustable parameters defining the apparatus configuration. At each side of the experiment a dichotomic observable is measured, say $A(a)$ for the apparatus I_a and $B(b)$ for the apparatus II_b. The choice of the observables depends on the value of the *local* parameters a and b, each of the *local* observable is taken to have +1 and -1 as possible outcomes. The correlation function between the two observables is $C_\rho(a,b) = \langle A(a)B(b)\rangle_\rho$, in the hypothesis of local realism the following inequalities hold:

$$B = |C_\rho(a,b) + C_\rho(a,c) + C_\rho(d,b) - C_\rho(d,c)| \leqslant 2 \tag{11}$$

for any value of the parameters a, b, c, d and any ρ.

In order to describe these inequalities from the point of view of the tomographic representation, we define an associated matrix in terms of which the inequalities (11) can be written, eventually this matrix will turn to be a stochastic matrix. Let us first consider the simplest case of a bipartite system composed by two two-level systems. In order to deal with the generic case, we consider the unitary tomogram corresponding to the density matrix ρ:

$$\omega_\rho(m_1, m_2, a, b), \tag{12}$$

where a and b are short-hand notations for $u_1(a)$ and $u_2(b)$. Putting $m = 1$ and $m = -1$ respectively for polarization parallel and anti-parallel to the quantization direction we can define the following matrix:

$$M_\rho = \begin{bmatrix} \omega(1,1,a,b) & \omega(1,1,a,c) & \omega(1,1,d,b) & \omega(1,1,d,c) \\ \omega(1,-1,a,b) & \omega(1,-1,a,c) & \omega(1,-1,d,b) & \omega(1,-1,d,c) \\ \omega(-1,1,a,b) & \omega(-1,1,a,c) & \omega(-1,1,d,b) & \omega(-1,1,d,c) \\ \omega(-1,-1,a,b) & \omega(-1,-1,a,c) & \omega(-1,-1,d,b) & \omega(-1,-1,d,c) \end{bmatrix}. \tag{13}$$

Note that each column of this matrix is a well-defined probability distribution which corresponds to the tomogram with particular values of the parameters; hence M is a *stochastic matrix*. Thus a stochastic matrix is associated with a quantum tomogram in a way which is somehow analogous to the relation between density matrices and quantum maps [15–17]. Also note that the order in which the columns are organized with respect to the parameters a, b, c, d resembles the structure of a direct product. It is easy to check that for simply separable states the associated stochastic matrix factorizes as the direct product of two stochastic matrices each one corresponding to the one-particle tomogram:

$$\rho = \rho_1 \otimes \rho_2 \Rightarrow M = \begin{bmatrix} \omega_1(1,a) & \omega_1(1,d) \\ \omega_1(-1,a) & \omega_1(-1,d) \end{bmatrix} \otimes \begin{bmatrix} \omega_2(1,b) & \omega_2(1,c) \\ \omega_2(-1,b) & \omega_2(-1,c) \end{bmatrix}. \tag{14}$$

That is, a simply separable state corresponds to a factorized stochastic matrix. Analogously, a separable state corresponds to a stochastic matrix which is the convex sum of the factorized stochastic matrices.

With the labeling $m = -1, 1$ the discrete index in the tomogram is just the value of the relevant observable, so the expectation value for the correlation is simply written as $C(u_1, u_2) = \sum_{m_1,m_2} m_1 m_2 \omega(m_1, m_2, u_1, u_2)$. Introducing the matrix

$$I = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \end{bmatrix} \tag{15}$$

the CHSH inequalities (11) can be written in the following way:

$$B = |\mathrm{tr}(IM)| \leqslant 2. \tag{16}$$

This expression will be used in the following sections where we define, by means of the machinery of the *qubit portraits* introduced in [4], a stochastic matrix in the case of a bi-partite system composed of two qudits.

4. Qubit portraits of qudit systems

In this section we consider the CHSH inequalities in the case of a system composed of two qudits. In order to do this one needs to define a couple of dichotomic observables and to study the correlations between them. This discussion belongs to a general setting made of a system composed of two (d-dimensional) subsystems; in each of one, two local observables are measured and each measurement has two possible outcomes. While in the qubit case any non-trivial observable can be associated with a dichotomic observable with outcomes +1 and -1; this is not the case for the qudit systems in which dichotomic observables do not represent the generic case. This kind of problem was already considered in [18]; in the present work we exploit the machinery introduced in [4] which allows one to define a family of probability distributions which mimic a qubit tomogram and give a complete description of a qudit system; this kind of representation is called a *qubit portrait*.

As we have already recalled, the tomogram of a quantum state is a family of probability distributions over all possible measurement outcomes in a given basis, where each measurement outcome corresponds to a one-dimensional projector $P(m) = |m\rangle\langle m|$. In the same way one can consider a two-dimensional or in general an n-dimensional projector defined as $P(m_0, m_1, \ldots, m_{n-1}) = \sum_{k=0}^{n-1} |m_k\rangle\langle m_k|$ and consider the corresponding probability. Since the projectors on the basis vectors are orthogonal to each other, in the tomographic representation this probability is given by the sum over independent events $\sum_k \omega(m_k, u)$. As an example, let us consider the case of a qutrit system. In this case we have a unitary tomogram $\omega(m, u)$ where $m = 0, 1, 2$ and $u \in \mathrm{SU}(3)$. Identifying the *events* $m = 0$ and $m = 1$, we can define a *qubit portrait* of the qutrit state as the family of probability distribution $\omega'(m', u)$, with $m' = 0, 1$ and $\omega'(0, u) = \omega(0, u) + \omega(1, u)$ and $\omega'(1, u) = \omega(2, u)$. Analogously, one can define other two qubit portraits of the qutrit state. In the same way we can reduce any qudit tomogram to a family of probability distributions over a dichotomic variable and so define a qubit-portraits representation for any qudit tomogram. The same considerations can be extended to the case of spin tomography and to the case of multipartite systems: for instance, a tomogram for the state of a system composed of two qutrits can be reduced to a family of probability distributions over two dichotomic variables, which corresponds to a two-qubit portrait of the two-qutrit system. In this fashion one can define, as in the previous section, a (square) stochastic matrix using the qubit portraits of a qudit–qudit system.

5. Qubit portraits of qutrit states and CHSH inequalities

In this section we study the CHSH inequalities applied to the case of qutrit–qutrit system, in particular we focalize our attention onto the families of Werner states [19] and isotropic states [20]. In order to define a dichotomic variable, we reduce the qudit states to a family of probability distributions which are the corresponding qubit portraits. This yields to identifying a stochastic matrix, analogous to the one presented in section 3, which is defined by means of the qubit portrait. Having reduced the qudit–qudit system to an *effective* qubit–qubit system, we can consider the inequalities (16). In principle one can write several inequalities (not all independent) which correspond to all the possible qubit portraits that can be defined starting from the given qudits tomogram. In the case of the qubit–qubit system, the CHSH inequalities have been already considered in the tomographic picture in [18], and now we consider the inequalities (16) defined with the help of qubit-portraits machinery.

The first family is given by the qudit–qudit Werner states, which is a one-parameter family of quantum states defined as

$$W = (d^3 - d)^{-1} \left[(d - \phi)\mathbb{I} + (d\phi - 1)\mathbb{V}\right] \tag{17}$$

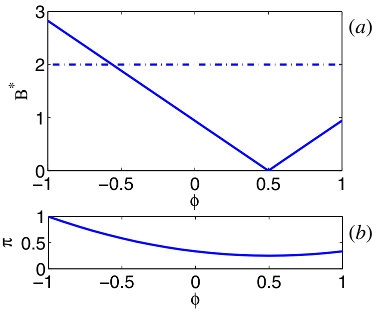

Figure 1. (*a*) Maximum value of the Bell number (16) for a two-qubit Werner state (17) as a function of the state parameter (solid line) compared with the maximum value allowed by local hidden variables theories (dot-dashed line). (*b*) Purity of the two-qubit Werner state.

for $\phi \in [-1, 1]$, where $\mathbb{I}$ is the identity operator in the two-qudit space and $\mathbb{V}$ is the *flip operator* defined as $\mathbb{V}|\psi_1\rangle|\psi_2\rangle = |\psi_2\rangle|\psi_1\rangle$. The state (17) is separable for $\phi \geqslant 0$ and entangled otherwise. The second family is given by the isotropic states:

$$S = (d^2 - 1)^{-1}[(1 - p)\mathbb{I} + (pd^2 - 1)|\psi\rangle\langle\psi|] \tag{18}$$

for $p \in [0, 1]$, where $|\psi\rangle = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} |ii\rangle$ is a maximally entangled state. The state (18) is separable for $p \leqslant d^{-1}$ and entangled otherwise. Note that, for $d = 2$ and $p \geqslant 0$, the two families are related by a reparametrization and a partial transposition.

We also need to specify what kind of tomogram we want to use, for the sake of simplicity we restrict our discussion to the case of polarization measurements; this is the case in which we take the subgroup of the unitary group U(d) given by an irreducible representation of SU(2) and consider the local spin tomogram. In this case the parameters defining the local observables are just a pair of Euler angles which identify the direction of polarization. In the two-qubit case, the local group is $\mathrm{SU}(2) \otimes \mathrm{SU}(2)$ acting on $\mathbb{C}^2 \otimes \mathbb{C}^2$, when the qubit portraits arise from qutrit one should use $\mathrm{SU}(2) \subset \mathrm{SU}(3)$ acting irreducibly.

First of all, let us consider the case of qubits. In this case the construction of the qubit portraits is redundant and our discussion is just a different way to deal with CHSH inequalities, nevertheless this example can be an useful term of comparison with respect to higher dimensional non-trivial configurations. We have computed the spin tomogram of the two-qubit Werner states and computed the maximum of the quantity (16), denoted B^*. Note that the maximum is taken with respect to all the possible choices of local observables which in the case of spin tomography are identified by four unit vectors on the Bloch sphere $\hat{n}_a, \hat{n}_b, \hat{n}_c, \hat{n}_d$. The results are shown in figure 1(*a*). The same calculation has been done for the case of two-qubit isotropic states and the corresponding results are plotted in figure 2(*a*). The plots 1(*b*) and 2(*b*) show the purity $\pi = \operatorname{tr}\rho^2$ as a function of the parameter of the corresponding states.

For the case of a two-qutrit state, we have compared the results obtained with the qubit-portrait method with the qutrit–qutrit Bell's inequalities presented in [21] which generalize the CHSH inequalities. In our notation we can write them as

$$\begin{aligned} I_3 = \{&P[A(a) = B(b)] + P[A(c) = B(b) - 1] \\ &+ P[A(c) = B(d)] + P[A(a) = B(d)]\} \\ &- \{P[A(a) = B(b) - 1] + P[A(c) = B(b)] \\ &+ P[A(c) = B(d) - 1] + P[A(a) = B(d) + 1]\} \leqslant 2, \end{aligned} \tag{19}$$

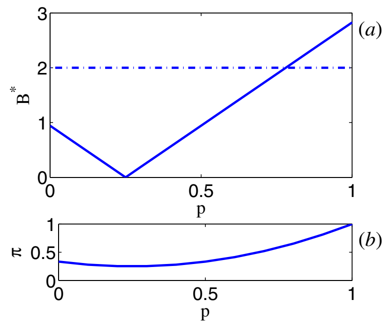

Figure 2. (*a*) Maximum value of the Bell number (16) for a two-qubit isotropic state (18) as a function of the state parameter (solid line) compared with the maximum value allowed by local hidden variables theories (dot-dashed line). (*b*) Purity of the two-qubit isotropic state.

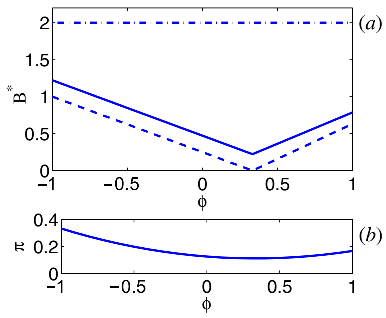

Figure 3. (*a*) Maximum value of the Bell number (16) for a two-qutrit Werner state (17) as a function of the state parameter (solid line) compared with the maximum value of I_3 from equation (19) (dashed line) and the maximum value allowed by local hidden variables theories (dot-dashed line). (*b*) Purity of the two-qutrit Werner state.

where

$$P([A(a) = B(b) + k] \equiv \sum_j \omega(j + k, j, a, b), \tag{20}$$

and the sum $j + k$ is modulo 3. Note that, also in this case, the inequalities can be written using the language of tomograms in a natural way.

For the two-qutrit Werner states, we have first considered all the possible two-qubit portraits which are computed by means of the procedure described in section (4). The maximum of the Bell number (16) is determined with respect to both polarization vectors which define the set of local observables and the different qubit portraits of the two-qutrit system. The results are plotted in figure 3(*a*) together with the maximum value of the analogous quantity I_3 from equation (19). The plot shows that the qubit-portraits method cannot reveal quantum correlations in two-qutrit Werner states. Analogously we have computed the maximum of (16) for the two-qutrit isotropic states; in this case, as shown in figure 4(*a*), the qubit-portrait method is able to witness the presence of quantum correlations. The results are plotted together with the maximum value of I_3 from equation (19).

The different capability of the CHSH inequalities approached with the machinery of the qubit portraits to recognize quantum correlations in Werner and isotropic state for $d = 2, 3$

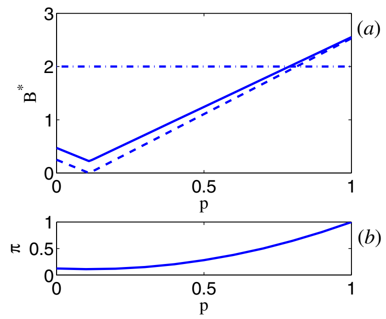

Figure 4. (*a*) Maximum value of the Bell number (16) for a two-qutrit isotropic state (18) as a function of the state parameter (solid line) compared with the maximum value of I_3 from equation (19) (dashed line) and the maximum value allowed by local hidden variables theories (dot-dashed line). (*b*) Purity of the two-qutrit isotropic state.

can be related to the different value of the purity of the corresponding states which are plotted in figures 3(*b*) and 4(*b*).

For the case of the two-qutrit isotropic states, the minimal value of the state parameter, arising from our method, that yields a violation of Bell's inequalities is $p^{B}_{\min} \simeq 0.7893$ for the CHSH inequalities and $p^{I_3}_{\min} \simeq 0.8139$ for the inequalities (19). In our notation, the singlet fraction is $q = (9p - 1)/8$, yielding $q^{B}_{\min} \simeq 0.7630$ and $q^{I_3}_{\min} \simeq 0.7906$. A comparison with the results presented in [21, 22] yields the observation that the local spin tomography, although based on an irreducible action of the group SU(2), cannot give a complete information about the violation of Bell inequalities. In other words, even though a tomographic set gives complete information about the quantum state and allows the reconstruction of the density operator, the local spin tomogram cannot necessary reach the configuration corresponding to the maximal violation of a Bell-like inequality.

6. Conclusions and outlook

In this paper we have further investigated the method of qubit portraits of qudit states first discussed in [4]. This method arises in a natural way in the tomographic description of quantum mechanics; it allows us to map a qudit tomogram onto a family of probability distributions which mimic a family of qubit tomograms. The method is applied in relation to the study of non-classical correlations in quantum systems, it allows a study of the CHSH inequalities for generic bipartite qudit systems.

Exploiting the tomographic approach to quantum mechanics, it is possible to associate a stochastic matrix to any bipartite quantum system with a finite number of levels, its structure is related to the structure of the CHSH inequalities and in term of it the presence of quantum correlations can be studied. Some examples have been presented regarding two special classes of bipartite states, namely Werner and isotropic states. The results show that performing the operation of the qubit portraits can lead to some loss of information about quantum correlations, as it is witnessed by the absence of violations of the CHSH inequalities in the case of the qubit portraits of two qutrits Werner states. On the other hand, the study of other two-qutrit Bell's inequalities with the framework of quantum tomography leads to the conclusion that even though the spin tomogram allows the reconstruction of the quantum state, it does not necessary provide the maximal violation of Bell-like inequalities.

Following [7, 17], in future publications we will consider possible extensions of the present work to the case of systems with higher dimensions and continuous variables. Other possible applications of the qubit-portraits method can be the study of other Bell-like inequalities which involve more than two choices of local observables per part.

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